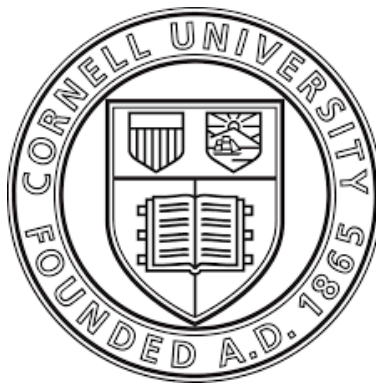


Modeling Countercurrent Arteriovenous Heat Exchange and Blood Flow in a Finger Exposed to Cold

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Keywords: countercurrent, finger, thermoregulation, heat exchange, blood flow



BEE 4530: Computer-Aided Engineering | Applications to Biological Processes

Cornell University

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1 Executive Summary

It is widely accepted among the scientific community that countercurrent heat exchange between blood vessels developed as an advantageous evolutionary trait for preserving body heat in humans and other animals; however, much of the theory behind this is qualitative in nature^{4,5}. In this study, we provide a quantitative model of countercurrent heat exchange in a human finger exposed to freezing temperatures. We seek to compare the impact of changes in various physiologically-based parameters on convective heat loss from the finger. By comparing the net effects of several parameters to that of countercurrent heat exchange, we can make a concrete claim about how important the presence of countercurrent exchange as an evolutionary trait is to the preservation of core body temperature.

In this study, we consider the tissue temperature within a human finger that is losing heat to the surrounding cold blowing air, while also gaining heat from blood vessels that participate in countercurrent heat exchange. To investigate the mechanism of countercurrent heat exchange in a human finger, we used COMSOL, a multiphysics finite element analysis and simulation software, to develop a simple geometry and replicate the heat exchanging properties of the human finger when exposed to extreme cold or freezing external conditions. The two major blood vessels we consider will be separated and surrounded by finger tissue that has its own physical properties. Heat is transferred from the flowing blood, through the finger tissue, and is lost to the surrounding cold environment. Using this model, we were able to analyze how the blood and tissue temperature gradients change with position and time over while exposed to external freezing conditions.

The model demonstrated the advantages of countercurrent exchange as a critical trait in mediating convective heat loss in an organism, thereby supporting the accepted idea that it evolved as a mechanism to preserve heat. In transferring much of the heat from the arterial blood to the venous blood, instead of to the tissue, the body retains that thermal energy as opposed to allowing it to escape as convective heat loss. The net effect is a decrease in the average tissue temperature, which equates to a decrease in the flux of outward heat, henceforth maintaining core body temperature at the expense of that of the extremities.

The results of this study provide insight on the relative importance of different physiological responses to the cold. While other physiological adaptations may serve to mitigate convective heat loss and therein preserve core body temperature, the most significant factor affecting heat loss was the distance between vessels. Thus, consistent with qualitative assessments, the evolution of blood vessel geometries has proven mathematically to play a critical role in the conservation of body heat. By understanding the mechanistic underpinnings of how this works, we can form a better basis of understanding for future bio-inspired designs.

2 Introduction

Blood flow throughout warm-blooded animals is a major factor in their abilities to maintain sufficient body warmth, particularly in the presence of external freezing conditions⁸. Figure 1 depicts the orientations of the major arteries and veins in a typical human index finger, clearly showing that arteries and veins are located close to one another, oftentimes in pairs. Warm blood from the core of the body carries heat through arteries to be dissipated in the cool extremities. Upon return from the extremities, the blood carries some of this heat back towards the core. By engaging in countercurrent heat exchange, the temperature of the blood that is returning to the core is at a higher temperature than it would be if the vessels were flowing co-currently. For this reason, it is widely accepted that arterio-venous countercurrent heat exchange is employed by organisms to minimize heat loss through the extremities and maximize the heat being carried by the blood back to the body core¹². However, while some papers have attempted to perform a mathematical analysis of countercurrent exchange in human extremities, the results have been limited by the technology available at the time and, on a whole, do not serve to effectively analyze the magnitude of the role of countercurrent heat exchange in the preservation of thermal energy^{12,7,9}.

Mechanistically speaking, countercurrent exchange functions to conserve heat by facilitating the transfer of outgoing heat from the arterial blood to the incoming venous blood. Since the heat flux between vessels is a function of the temperature gradient in the intermediate tissue, factors that increase this gradient serve to improve heat retention, and factors which decrease it result in more heat loss¹².

In addition to countercurrent exchange, organisms have developed many other physiological adaptations to conserve heat and prevent hypothermia⁵. Vasodilation and vasoconstriction alter vessel diameter so as to increase or decrease the surface over which convective heat transfer from the blood to the tissue can occur¹⁸. Changing blood velocity affects the rate at which heat is delivered to the tissue^{2,3}. Lastly, the amount of insulating tissue between the vessels and the skin surface affects how quickly heat is conducted towards the skin⁸.



Figure 1: Image showing the locations of arteries and veins in a human finger. Arteries are displayed in red and veins are shown in blue. It should be noted that all vessels are positioned such that they run countercurrent to an adjacent vessel. Blood Vessel Geometries, Retrieved 5/6/2018 from <https://www.dreamstime.com/>

2.1 Problem Statement

Given the complex nature of the problem, it is difficult to make a claim regarding the importance of countercurrent exchange specifically in mitigating heat loss without numerical data to support the conclusions. In more concrete terms, it is hard to say whether vessel geometry evolved such that they run countercurrent to one another because increasing exchange between vessels is critical to heat retention, or simply because burying vessels deep in the tissue, in order to maximize insulation, forces them to necessarily run adjacent to one another. In modeling this process, we hope to:

1. Compute sensitivity of convective heat loss to physiological response factors such as vasodilation, vasoconstriction, and increases or decreases in blood flow.
2. Determine in isolation the sensitivity of convective heat loss to the distance between blood vessels in order to analyze the effects of increasing or decreasing countercurrent heat exchange between them. By lessening the distance between vessels, we increase heat exchange by increasing their view factor.
3. Determine in isolation the sensitivity of convective heat loss to the distance of the blood vessels from the edge of the finger. In doing so, we will analyze the effects of burying vessels deeper within tissue, thereby increasing the insulation in the finger.
4. Compare the relative sensitivities of heat loss from the finger to physiological responses, distance between vessels, and distance from edge of the finger to make a concrete claim about which factor is the most important in mitigating heat loss from the body.

2.2 Design Objectives

It is impossible to build a perfectly accurate model of a human finger. Besides variation between individuals, the vascular network of the finger is incredibly complicated and difficult to model. Thus, a heavily simplified model must be developed that preserves the important aspects of the physics in which we are interested. In developing a model of this process, our goals are to:

1. Develop a model that can be used to compute a temperature profile in the finger that is, on average, consistent with reported experimental values.
2. Build the model such that the effects on temperature profile of changes in the parameters listed above are representative of the effects in a live finger.
3. Report convective heat fluxes as a function of average surface temperature, which we will compute using our model in COMSOL.

3 Schematic of Geometry

To meet our design objectives, we simplified the finger to a cylinder in which a single artery and single vein run in opposing directions near the center of the digit. To simulate countercurrent exchange between the warm blood flowing in the finger's arteries and the cooler blood in the finger's veins, we combined each type of vessel and represent the veins collectively as one cylinder and the arteries as a separate cylinder, as was done in Shitzer's numerical analysis of countercurrent heat exchange⁹.

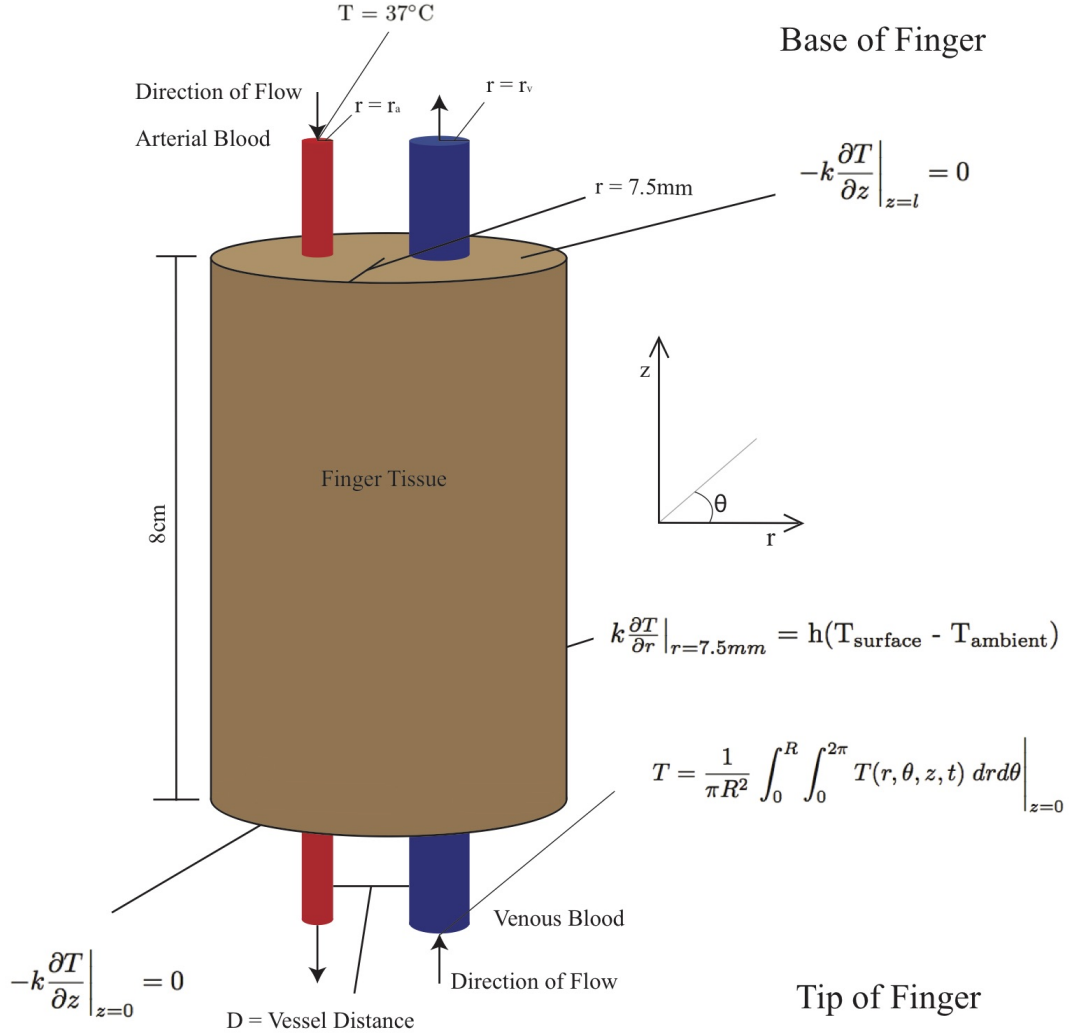


Figure 2: Schematic of simplified modeled finger, including dimensions, governing equations, and boundary conditions. The artery inlet temperature was set to equal 37°C , and the vein inlet temperature was set to equal the average temperature of the tissue surrounding the vein inlet. A convective heat flux boundary condition was set along the outer surface of the finger, using a convection coefficient of $10 \frac{\text{W}}{\text{m}^2\text{K}}$. At the transverse planes of the base and tip of the finger, the heat flux was set to equal zero.

4 Governing Equation and Boundary Conditions

The physics incorporated into this model are heat transfer in solids, heat transfer in fluids, and laminar fluid flow. It is thus governed by the heat equation and the Navier-Stokes equation for fluid flow:

4.1 Heat Transfer

Heat Equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} = \frac{k}{\rho C_p} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \left(\frac{\partial^2 T}{\partial z^2} \right) + \frac{1}{r^2} \left(\frac{\partial^2 T}{\partial \theta^2} \right) \right] \quad (1)$$

Boundary and initial conditions:

1. $-k \frac{\partial T}{\partial z} \Big|_{z=l} = 0$ at the interface with the hand, except for the arterial blood inlet.
2. $-k \frac{\partial T}{\partial z} \Big|_{z=0} = 0$ at the tip of the finger, except for the venous blood inlet.
3. $-k \frac{\partial T}{\partial r} \Big|_{r=7.5mm} = h(T_{r=7.5mm} - T_{\text{ambient}})$ over the rest of the finger surface.
4. $T_{inlet,venous} = \frac{1}{\pi R^2} \int_0^R \int_0^{2\pi} T(r, \theta, z, t) dr d\theta \Big|_{z=0}$ at the vein inlet.
5. $T_{inlet,arterial} = 37^\circ \text{C}$.

4.2 Fluid Flow

Navier Stokes

$$\rho \left(\frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla P + \mu \nabla^2 u \quad (2)$$

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (3)$$

Since the fluid flow is simply laminar flow through pipes, this simplifies to the velocity profile of fully developed laminar flow with a no slip condition at the surface:

$$\begin{aligned} u(r) &= -\frac{R^2}{4\mu} \left(\frac{\partial P}{\partial x} \left(1 - \frac{r^2}{R^2} \right) \right) \\ u(r) &= 2V_{\text{avg}} \left(1 - \frac{r^2}{R^2} \right) \end{aligned} \quad (4)$$

with inlet velocities specified as:

1. $V_{avg,arterial} = 10 \text{ cm/s}$
2. $V_{avg,venous} = 4 \text{ cm/s}$

and outlet pressures specified as the average absolute venous and arterial pressures. It should be noted that we modeled continuous flow rather than pulsating flow to reduce computational time significantly. A summary of our input parameter values and variable definitions can be seen below, in Tables 1 and 2.

Table 1: Input Parameters and Variable Definitions

Parameter	Symbol	Finger Tissue	Blood	Units
Thermal Conductivity ¹¹	k	0.35	0.45	$\frac{W}{m \cdot K}$
Density ¹¹	ρ	1060	1060	$\frac{kg}{m^3}$
Specific Heat ¹¹	C_p	3900	3900	$\frac{J}{kgK}$
Viscosity ⁶	μ	–	0.0028	$Pa \cdot s$

Table 2: Input Parameter Values

Parameter	Symbol	Value	Units
Ambient Temperature	$T_{ambient}$	0	$^{\circ}C$
Initial Tissue Temperature	$T_{initial, tissue}$	37	$^{\circ}C$
Initial Blood Temperature	$T_{initial, blood}$	37	$^{\circ}C$
Artery Inlet Temperature	$T_{inlet, arterial}$	37	$^{\circ}C$
Vein Inlet Temperature	$T_{inlet, venous}$	–	$^{\circ}C$
Arterial Blood Velocity ²	u_a	10	$\frac{cm}{s}$
Venous Blood Velocity ²	u_v	4	$\frac{cm}{s}$
Finger Radius ⁹	R	7.5	mm
Finger Length ⁹	l	80	mm
Absolute Arterial Pressure ⁶	P_a	1.1597	atm
Absolute Venous Pressure ⁶	P_v	1.1053	atm

5 System Discretization

An image of our mesh is shown below, in Figure 3. The extra fine mesh contained 410,059 elements.

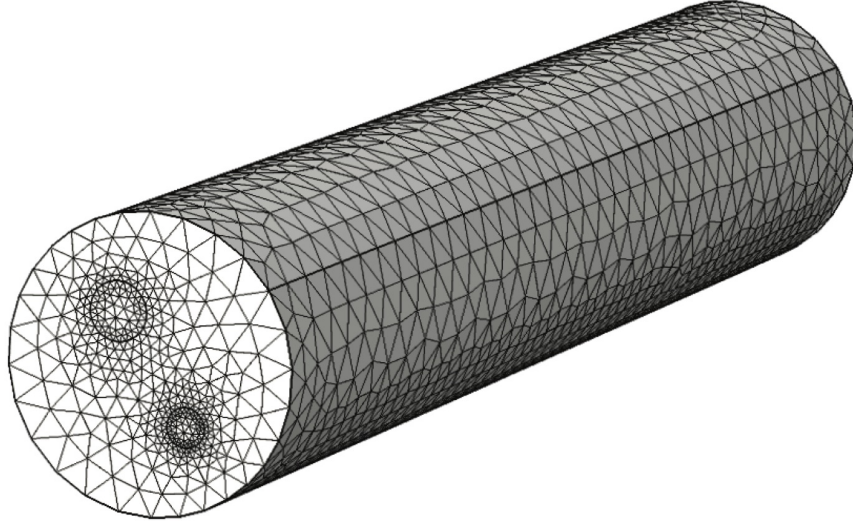


Figure 3: Mesh used for modeling in COMSOL.

To determine mesh convergence, the following parametric sweep was computed for coarse, normal, fine, finer, extra fine, and extremely fine physics-controlled meshes. The results of the parametric sweep varying the number of domain elements are shown below, in Figure 4.

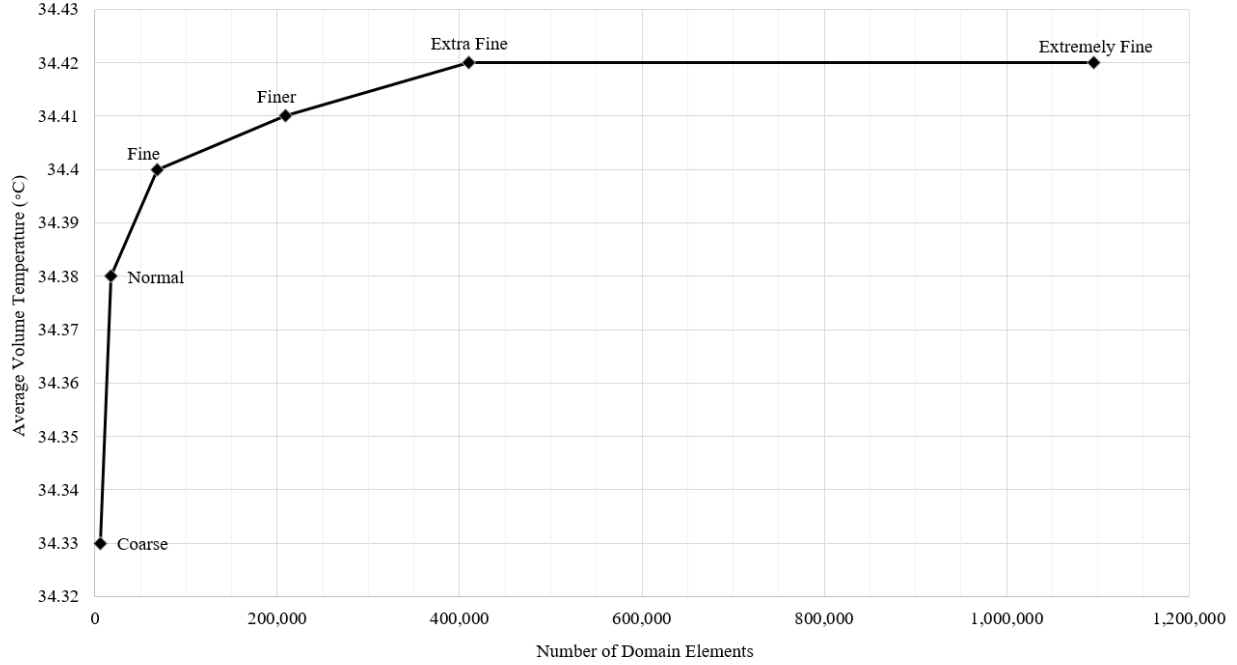


Figure 4: Mesh convergence plot. Volume averages were taken of finger temperature using extra coarse, coarse, fine, and extra fine meshes at 100 seconds. Convergence was observed after using an extra fine mesh containing 410,059 elements

The mesh convergence analysis used 6 different sized meshes with varying number of domain elements. For each mesh size, a volume average of the entire domain's temperature was taken at a time of 100 seconds, which was chosen because it falls in an area of transient temperature change. The analysis showed that despite the mesh size chosen, the volume average temperature showed little variability (less than 0.1 °C). Therefore we can conclude that our solution is nearly independent of mesh size. True convergence is observed after the use of an extra fine mesh; there is no discernible difference between extra fine and extremely fine meshes.

6 Preliminary Simulations

6.1 Temperature Profiles

Computing temperature over time across the domain with these conditions and mesh yielded the following temperature profiles at 15, 60, 120, and 1800 seconds, respectively. These results are shown in Figure 5.

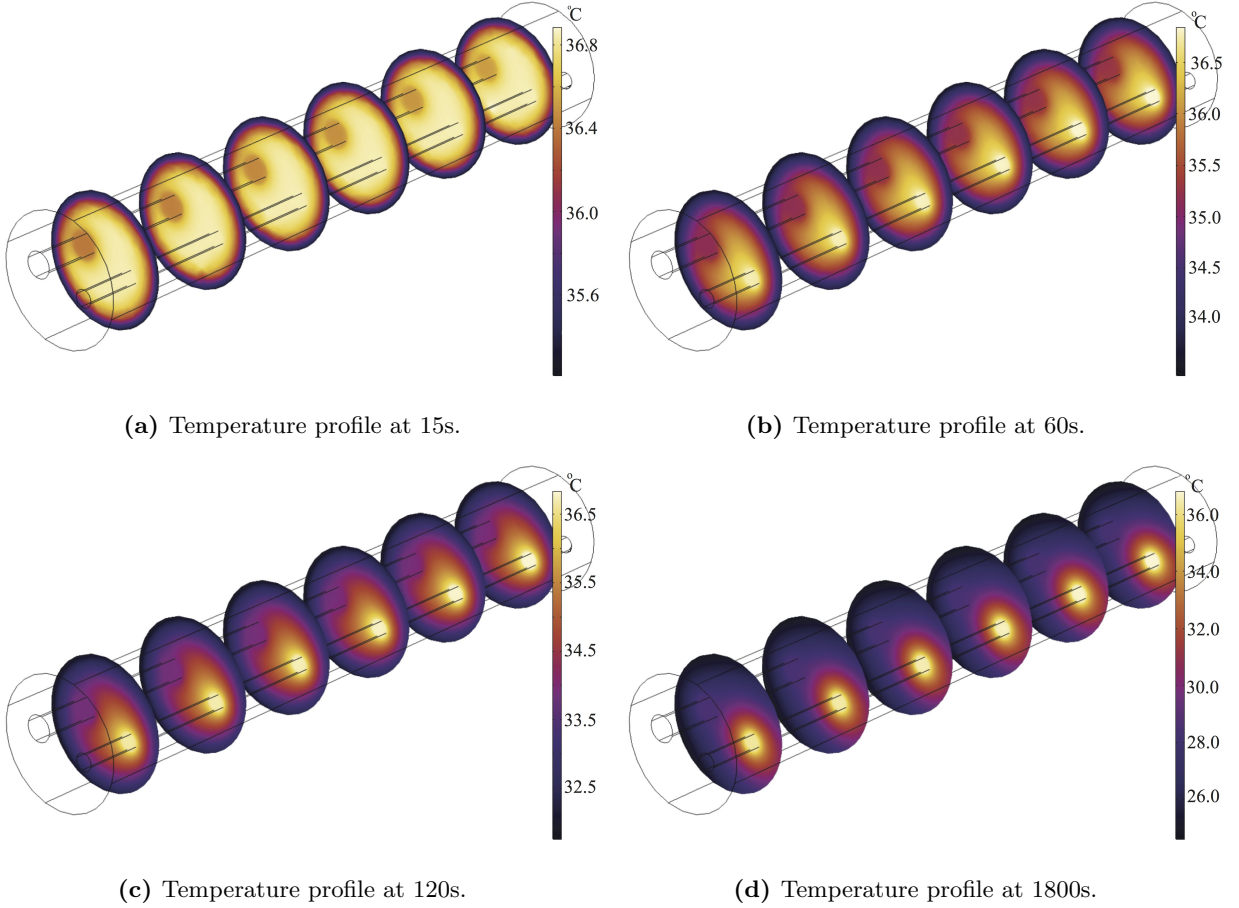
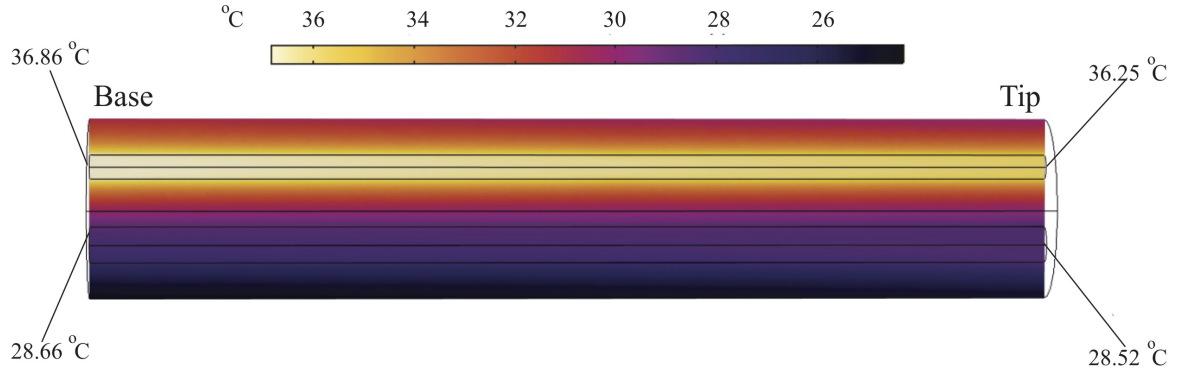


Figure 5: Temperature profiles in the finger at varying times.

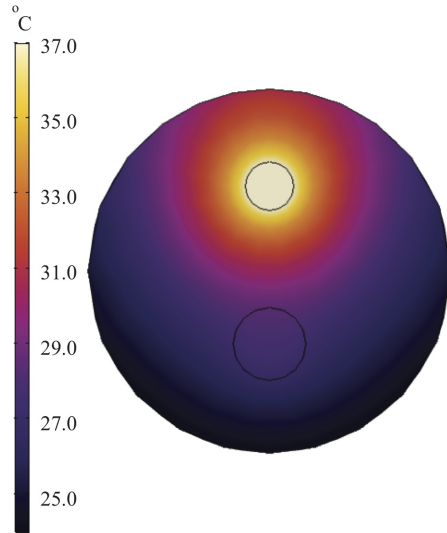
The items of note in this plot are that the finger is warmest around the artery and coldest around the vein, as is expected due to the difference in the vessels' inlet temperatures. Furthermore, there is a temperature gradient that develops along the length of the finger (z-direction), with the transverse plane nearest to the heart (base) being the hotter than the side that is farther (tip). This is due to the warm arterial blood losing heat to the surrounding tissue as it progresses along the finger towards the tip. To confirm that this gradient is a result of heat exchange between the vessels and not, we removed the flow term in the venous vessel and observed that no notable gradient develops along the length of the blood vessels.

6.2 Temperature Gradient

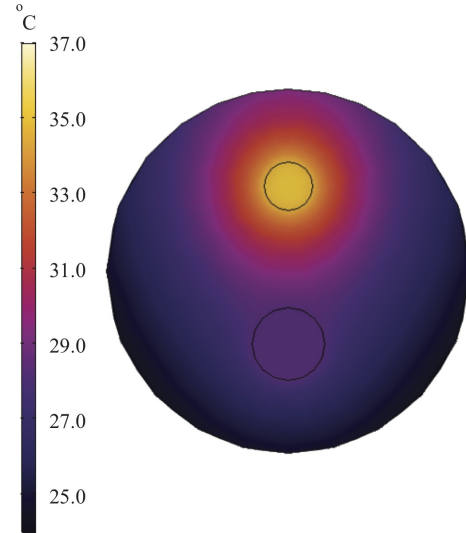
The aforementioned graded difference in temperature profile between the base and tip of the finger is even more visible when considering a slice along the finger in the axial direction, shown below in Figure 6. The arterial blood temperature decreases by approximately $0.6\text{ }^{\circ}\text{C}$ as it travels from the base of the finger to the tip; the venous blood temperature increases by $0.14\text{ }^{\circ}\text{C}$ while returning from the tip of the finger.



(a) Slice through the center of the finger.



(b) Profile at the base of the finger.



(c) Profile at the tip of the finger.

Figure 6: Temperature profiles in the finger after one hour. The distribution of temperatures in the tip of the finger is considerably lower than that of the base, further illustrating the aforementioned gradient.

As was argued previously in Figure 5, the average temperature is warmer in the side of the finger nearest

to the heart (base), than in the end cap (tip), showing the clear effect of this heat exchange that develops along the length of the finger.

It should lastly be noted that the finger cools with time, as expected. To show time-dependent change this more quantitatively, a plot of volume and surface average temperatures is shown below, in Figure 7.

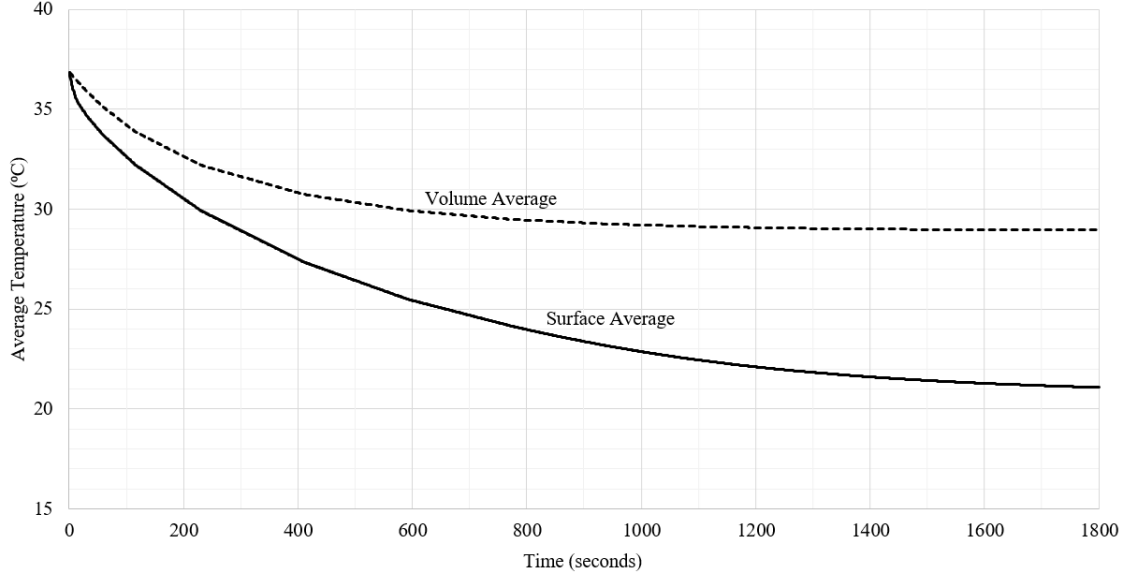


Figure 7: Volume and surface averages reported over the first 1800 seconds of computation. The transient phase of convective cooling in the finger takes place primarily in the first 1800 seconds.

The results shown in Figure 7 demonstrate that we can consider the transient phase of the problem to be the first 30 minutes. With respect to volume average in particular, very little deviation can be observed after around 20 minutes (1200 seconds). Since the interest of this study is the mitigation of convective heat loss, which is a function of surface temperature, surface average is the parameter of greatest interest to us; hence, we chose to use 1800 seconds as the interval of interest in our analyses. One can also see that surface average decreases more rapidly than volume average due to the high convective flux boundary condition imposed along the finger's outer surface.

7 Sensitivity Analysis

In order to better understand the factors that control convective heat loss, we conducted sensitivity analysis on various model parameters to determine how changes in these affect the surface temperature. The parameters we chose to analyze were the diameter of the vessels, the velocity of the blood, the displacement of vessels from the center of the finger, and the distance of the vessels from one another. These parameters were chosen to be representative of either physiological responses to hypothermic conditions, or geometric parameters regarding vessel location. The model parameters and their physical analogs are tabulated below for convenience.

Table 3: Model Parameters and Physical Significance

Model Parameter	Physical Analog
Diameter	Vasodilation and Vasoconstriction
Blood Velocity	Changes in Blood Perfusion
Displacement from Center of Finger	Depth of Vessels in Tissue
Distance Between Vessels	Extent of Countercurrent Exchange

7.1 Diameter

In order to simulate the effects of vasodilation and vasoconstriction, we first analyzed the sensitivity of this system to changes in vessel diameter. The changes in average convective heat flux from the finger surface over 1800 seconds for varying vessel diameters is shown in Figure 8. The basal level of average convective heat flux, obtained from our original model with arterial radius 1 mm and venous radius 1.5 mm, is $282.02 \frac{W}{m^2}$.

Changes in diameter result in minimal changes in convective heat loss. These are not negligible, but they are not substantial. At a maximum, vessel diameter effects approximately a 2% reduction or increase in convective heat loss. While 2% is still significant in the context of something as finely tuned as body temperature, there is not overwhelming sensitivity of convective heat flux to this parameter.

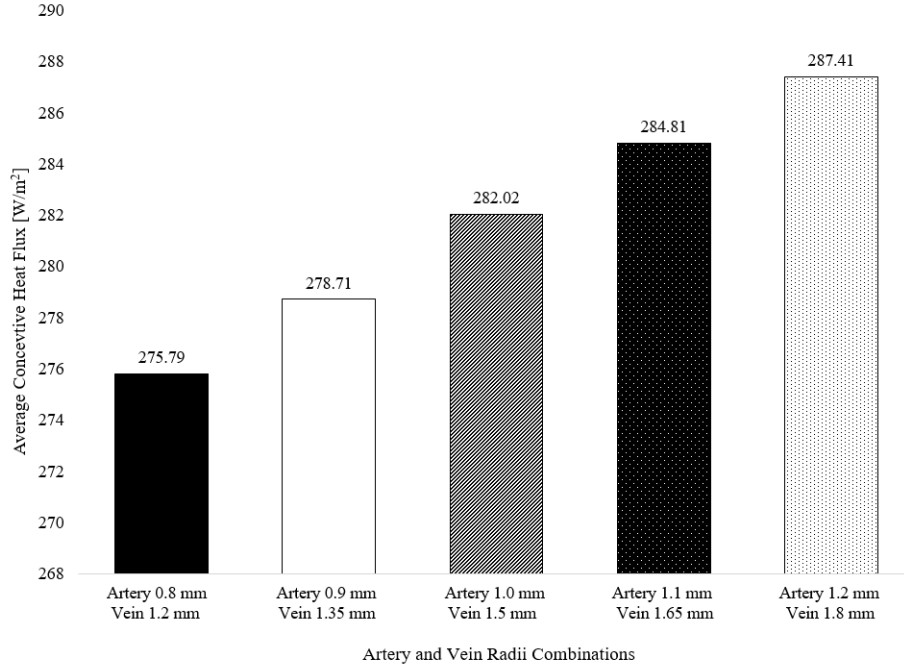


Figure 8: Sensitivity of average convective heat loss to blood vessel radius. Increasing the diameters of both blood vessels results in higher convective heat loss over the first 30 minutes, while smaller vessel geometries result in less convective heat loss from the finger.

7.2 Blood Velocity

The second parameter we considered was blood velocity. Varying velocities across the entire range of physiologically attainable arterial and venous velocities², we observed, again, minimal sensitivity to this parameter. These results are shown below in Figure 9.

Once again, we observe only about a 2% decrease in heat loss when velocity is decreased, and only a minuscule 0.4% increase in heat loss when velocity was increased to the maximum physically reported values². Similarly to the case of diameter, while these small changes may be significant in the context of body temperature, they demonstrate little sensitivity to velocity as a parameter.

One item of interest is that the model proves to be more sensitive to decreases in velocity than increases. Physically, this can be explained by the relationship between convective and conductive heat transfer. Higher blood velocity results in a larger convection coefficient between the blood vessels and the surrounding tissue. As the convection coefficient approaches infinity, brought about by a high velocity at the boundary, the time for heat to dissipate from the system becomes entirely governed by conductive, rather than convective heat transfer. Thus, for a system in which convection predominates, decreasing velocity has a far greater effect than increasing it.

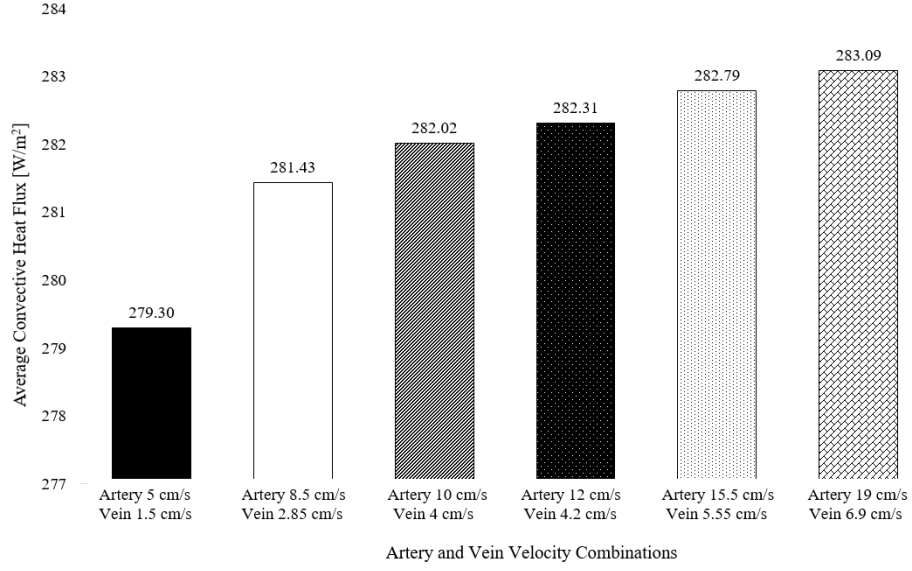
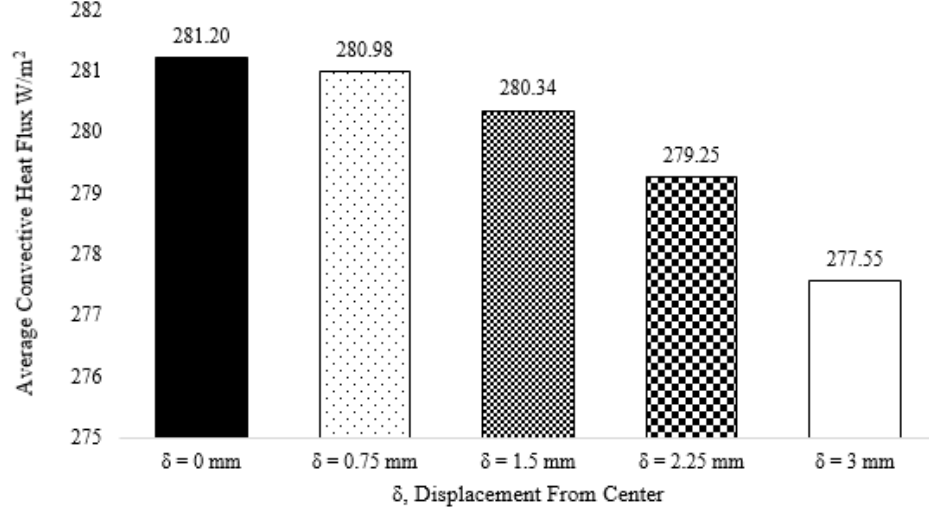


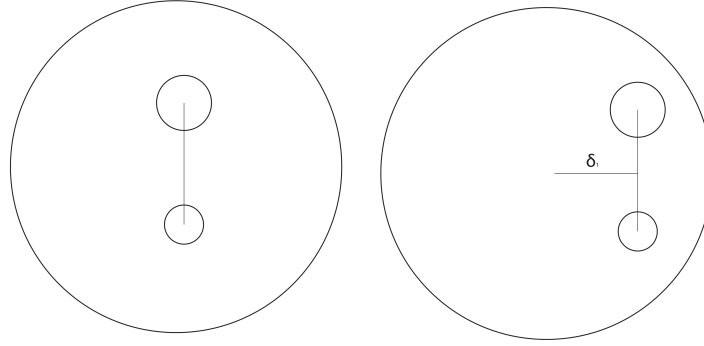
Figure 9: Sensitivity of convective heat loss to changes in blood velocities. Results indicate that convective heat loss decreases with decreasing velocity. Large decreases in blood velocities lead to a significant reduction in convective heat loss. However, increasing blood velocity by the same amount does not result in the same magnitude of increase in convective heat loss. Convective heat loss is sensitive to large decreases in blood velocity.

7.3 Displacement from Center

In order to consider the effect of burying vessels deeper within the finger, we conducted sensitivity analysis on the effect of displacing the vessels from center in the finger. A schematic showing how this was done is shown below in Figure 10b.



(a) Convective heat loss obtained for various distances of vessels from finger edge. As vessels are moved closer to the surface, convective heat loss decreases.



(b) Schematic showing the orientations of the vessels. δ represents the distance between the midpoint of vessel centers and the geometric center of the finger. In varying only δ , we alter the displacement of the vessels from the center but maintain vessel distance from each other, allowing us to study vessel displacement in isolation.

Figure 10: Sensitivity of convective heat loss to displacement of vessels from center of finger.

The trend shown by this data is actually contrary to what we expected. The data suggests that vessels situated further to one side of the finger actually result in less convective heat loss than if they were centrally located. This is particularly surprising because it implies that decreasing the amount of insulating tissue is not the most significant factor in the relationship between vessel location and heat loss. Rather, the results of this study suggest that decreasing the view factor (See Appendix C) of the vessel and the nearest surface proves more important. Perhaps a simpler way of understanding this is that the hottest the surface can

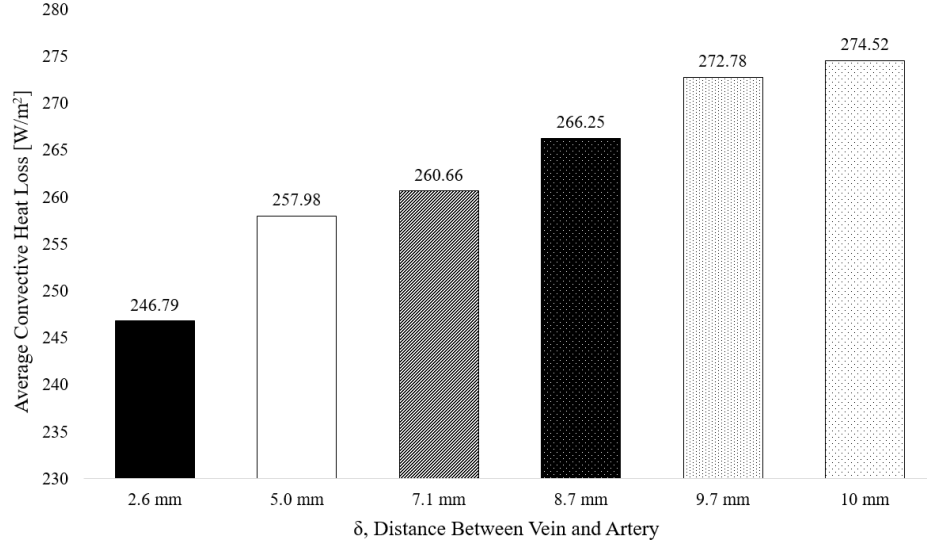
possibly get is the core body temperature, or 37°C . In contrast, the coldest side of the finger can decrease in temperature to a theoretical minimum of the ambient air temperature, or 0°C . Since the area of surface that sees a decrease in temperature as a result of displacing the blood vessels is far larger than the area that sees an increase, the benefits of increasing insulation across the majority of views between vessel and surface outweigh the, albeit greater, decrease in insulation for a few.

7.4 Distance Between Vessels

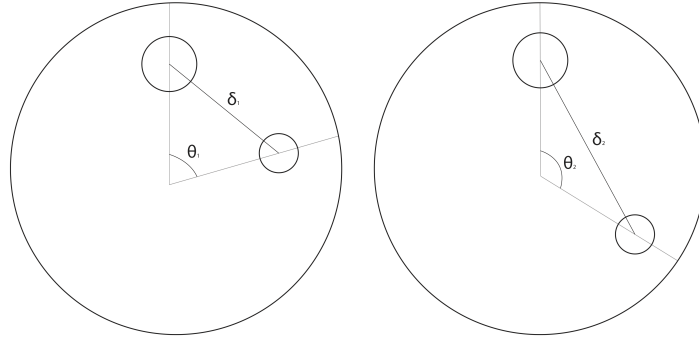
We also observe the effects of varying the distance between blood vessels. We kept the distance from the vein and artery's centers to the surface of the finger constant, and rotated the artery from 180° to 30° by 30° increments: 180° , 150° , 120° , 90° , 60° and 30° . This manipulation is visually explained in Figure 11b, which shows how the variation in rotation angle relates to the distance between the blood vessels. We collected the average surface temperatures across 1800 seconds using each of these configurations, and used these values to calculate the average convective heat flux across 1800 seconds. Figure 11a depicts these results for each of the 6 different artery positions. For convenience, configurations are denominated by the distance between centers of the vessels, which is computed as a function of the angle of displacement.

The results of this sensitivity analysis to distance between blood vessels indicate that less heat is lost from the finger when the vessels are located closer together. Because distance between the blood vessels is directly related to how much heat exchange they partake in; this result also has implications about the effect that countercurrent blood flow has on the preservation of body heat. Convective heat loss from the finger was lowest when the vessels were in their closest configuration. This significant result helps answer the question that prompted this project from the beginning: how important is countercurrent heat exchange in the preservation of body heat? Based on our findings, we conclude that countercurrent blood flow does indeed play a role in conserving thermal energy.

The steepest change in average convective heat loss is between the configurations in which δ is the smallest. Closely positioned vessels retain the most thermal energy by shuttling heat from one to the other, as opposed to allowing it to permeate throughout the volume of the finger and be lost through convective transfer to the surroundings. It should also be noted that as distance increases, the effects of this displacement become smaller. Logically this makes sense because when considering cylindrical geometries, the view factor between vessels (see Appendix C) decreases exponentially as distance between the vessels increases. Hence, conductive heat transfer in the tissue between the two vessels decreases at a proportional rate.



(a) Convective heat loss for different configurations of vessel geometry. As vessels are moved further apart, convective heat loss increases.



(b) Schematic showing the orientations of the vessels. δ represents the distance between the axial centers of the two vessels, while θ is the angle formed by the two centers and the center of the finger itself. In varying theta and delta, we alter the distance between the vessels but maintain vessel distance from the finger edge to study sensitivity to vessel distance in isolation.

Figure 11: Sensitivity of convective heat loss to distance between blood vessels.

7.5 Relative Importances of Parameters Through Sensitivity Analysis

To compare the four different sensitivities we decided to plot a normalized percent change in average convective heat flux for the lower and upper sensitivities tested.

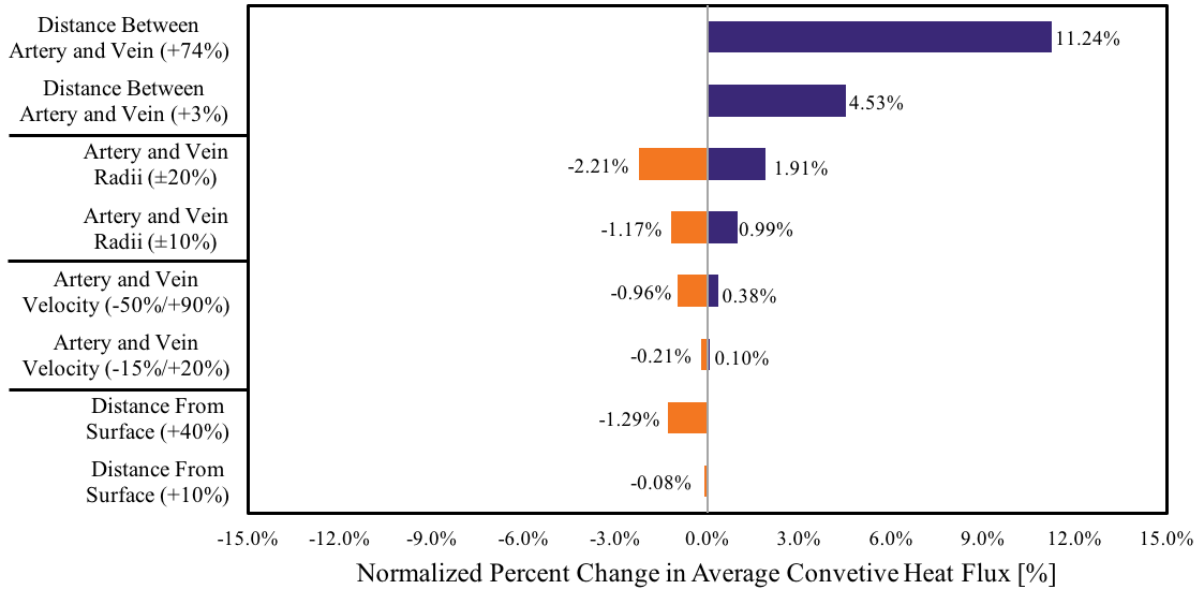


Figure 12: Normalized percent change in average convective heat flux after 30 minutes for each of the sensitivity parameters observed: distance between artery and vein, artery and vein radii, artery and vein velocity and distance from surface.

The parameter to which heat loss is most sensitive is the distance between the artery and vein. The change from 10 to 9.7 mm (a 3% change in displacement) yielded a 4.53% change to the average convective heat flux from the finger. This trend was consistent, with the closest displacement change from 10 mm to 2.6 mm (a 74% change in displacement) yielding a 11.24% change to the average convective heat flux. Artery and vein velocity was the parameter to which heat loss was least sensitive, with a arterial velocity change of -50%/+90% yielding only a -0.96% and 0.38% change, respectively. As noted earlier, we are less concerned about the actual values that COMSOL computes for these sensitivity analyses, but rather the trends associated with them. This helps to show the importance of countercurrent heat exchange design and explain the physiology of why arteries and veins run in close proximity to each other and in opposite directions throughout your body.

8 Validation

In order to validate our model, we compared the results of a physical experiment conducted by Shitzer et al. to values obtained using analogous inputs in our computational model¹⁰. In Shitzer’s experiment, seven human subjects had one finger’s temperature and blood perfusion monitored across one hour while the hand was exposed to controlled 0°C air. For each subject, the temperature and blood perfusion were monitored at a point directly under the fingernail bed. In our comparative analysis, we measured temperature values at a point in a similar location, 1 mm below the skin’s surface at the tip of the finger domain (the point $x = 79$ mm, $y = 7.4$ mm, $z = 0$ mm in COMSOL). Shitzer’s results of measured blood perfusion and temperature across 1 hour are presented as ranges: the average temperature plus and minus one standard deviation. We used these measured blood perfusion values to calculate high, low, and average velocities for each subject that correspond to the size of each of our model’s blood vessels. After running a parametric sweep across all twenty-one velocity values, we plotted the temperatures obtained at the specified point in the fingertip after one hour using each subject’s calculated upper bound, average, and lower bound blood velocity values. Shown in Figure 13, we plotted our model’s computed temperature values for each subject’s fingertip temperature adjacent to the experimental temperature values obtained by Shitzer.

As can be seen in Figure 13, the temperature ranges measured during Shitzer’s experiments overlap with the temperature ranges computed using our model for five of the seven subjects. The two subjects whose computed fingertip temperature ranges do not at all overlap with the experimental results (subjects 3 and 6) both had high blood perfusion values that were extreme outliers compared to the other five subjects. While the average blood perfusions measured in subjects 3 and 6 were approximately 17 and 11 milliliters of blood per milliliter of tissue per hour, respectively, the average blood perfusion among the other five subjects was only 3.75 milliliters of blood per milliliter of tissue per hour. When used in our computational model, these extreme blood perfusion values translated into abnormally high velocity values in the blood vessels, and as a result, the computed temperatures in these subjects’ fingertips were significantly higher than what was measured. Shitzer discusses that these cases of high blood perfusion may be due to a dramatic physiological response elicited in each of the two outlier subjects after their hand was exposed to freezing cold air for one hour¹⁰. Natural responses to environmental changes such as this vary among all people, and so we agree with Shitzer’s attribution of these outliers to differences in the subjects’ physiological responses to the cold. We conclude that our model is valid for individuals not exhibiting extreme physiological responses to the cold.

This comparative analysis suggests that our model may be too simplified to be suitable for obtaining

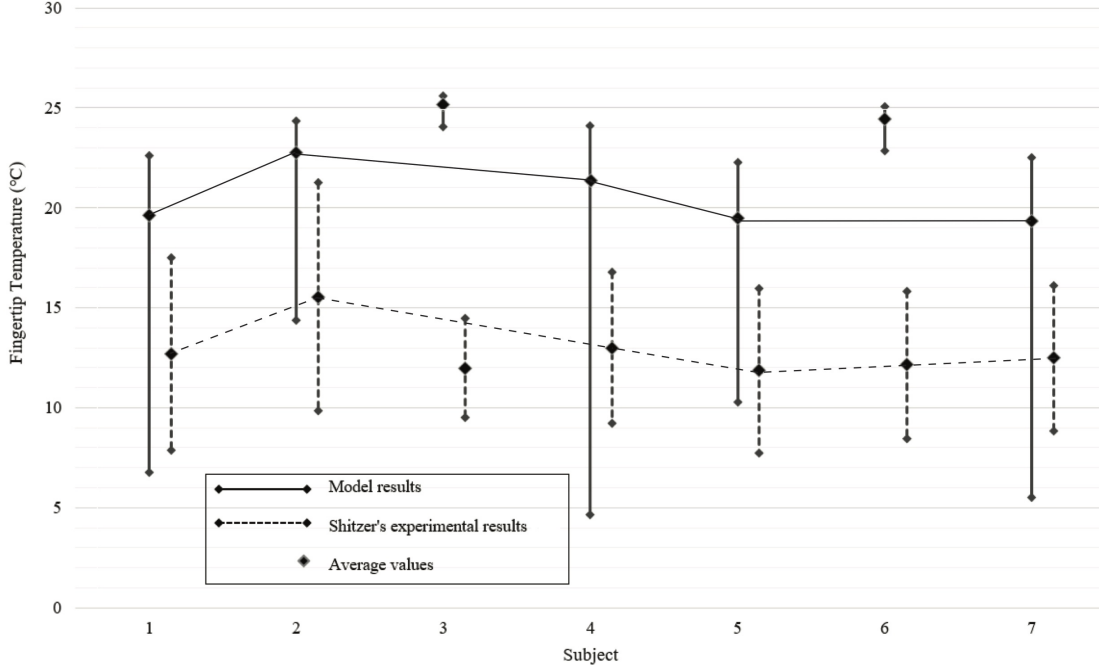


Figure 13: Comparison of our model results with experimental results from Shitzer et al.¹⁰. In his experiment, Shitzer simultaneously measured temperature and blood perfusion in a fingertip of seven subjects as their hand was exposed to 0°C air, and reported the average fingertip temperatures and blood perfusions for each subject after one hour, plus and minus one standard deviation. After converting each subject's blood perfusion values into upper bound, average, and lower bound blood velocities corresponding to vessels of our model's size, we ran our model using each of these blood velocities and recoded the resulting temperature at a point in the tip of the finger ($x = 79$ mm, $y = 7.4$ mm, $z = 0$ mm in COMSOL) after one hour. For each subject, we plotted the range of temperatures at this point computed using our model adjacent to the experimental results measured in Shitzer's study. The two outliers are individuals for whom abnormally high rates of blood perfusion were reported. The lines shown on the graph show the matching effects of velocity on experimental and numerical data when outliers are removed from the sample set.

accurate temperature values across time and location; however, it also validates our claim that our model is useful in determining the relative sensitivity of heat loss from the finger to various parameters. Our sensitivity analysis on blood velocity showed that heat loss is significantly more sensitive to decreases in velocity than it is to increases of the same magnitude. This is also evident from our comparative validation shown in Figure 13. For all seven subjects, the magnitude of the change in temperature that was induced when the velocity was decreased by one standard deviation (lower bound) was much greater than the magnitude of change induced by increasing the velocity by the same amount (upper bound). Furthermore, the general trends in fingertip temperature across the non-outlier subjects obtained by our model align with the trends seen in the experimental data. This can be seen visually by connecting the average temperature markers across each subject's model results and across each subject's experimental results, and comparing the shapes of the two resulting trend lines. Comparing these trend lines in Figure 13 shows that our model behaves similarly to the experimental results. Where one subject's experimentally measured temperature was higher than another

subject's, our model also predicted a higher fingertip temperature that is higher by approximately the same amount. For example, Subject 2's average fingertip temperature measured by Shitzer was slightly warmer than Subject 1's by approximately 3 degrees, and our model predicted the same difference between Subjects 1 and 2.

9 Discussion

9.1 Sensitivity

In order to claim that blood vessels evolved to run adjacent to one another so as to benefit from countercurrent heat exchange, it is necessary to demonstrate that:

1. Countercurrent exchange serves to preserve thermal energy and does so on at least the same order of magnitude as other physiological responses.
2. The benefit, in terms of heat conservation, of vessels running countercurrent to one another is comparable or greater than the benefit of burying vessels deep in the tissue to maximize insulation.

If other factors proved to be many times more significant and the benefits of geometric orientation of vessels were negligible, then there would be minimal evolutionary pressure to develop vasculature that benefited from countercurrent exchange. Similarly, if the impact of burying vessels deep within the tissue were considerably greater than that of vessels running countercurrent to one another, the adjacent locations of the vessels would more likely be a product of increasing insulation rather than maximizing exchange.

Contrary to both of these conditions, our results confirm that minimizing distance between vessels is the most important factor in decreasing heat flux through the skin. By studying vasodilation, vasoconstriction, increases and decreases in blood velocity, changes in distance from the surface, and changes in distance between the vessels, each in isolation, we show that the difference between closely and distantly situated vessels plays a greater role in heat conservation than any other factor does. Surprisingly, vessel proximity to the surface actually resulted in a decrease in heat loss, therein discrediting the idea that blood vessels are simply buried in the finger next to one another to maximize their depth and insulation.

Although it is not the focus of this study, an interesting addendum on the physics behind which vessel proximity to surface results in less convective heat loss. A proposed mechanism based off of view factors between vessels is discussed in Appendix C.

9.2 Implications

The implications of this study lie in understanding the selection factors by which evolution has designed today’s organisms. Conservation of thermal energy, while clearly beneficial in many ways, comes at a cost. Specifically, it is achieved by lowering the body temperature in the extremities so as to decrease the convective flux of heat from the skin. Doing so necessarily has negative consequences.

Warm blooded organisms function best when their cells are kept at a high temperature, regardless of what that temperature may be. Showing that evolution has selected for organisms that decrease the temperature of their extremities as a means of conserving heat implies that thermal efficiency is more important to survival than maximizing functionality of the appendages or limbs.

Similarly, in effecting a decrease in the time it takes to cool the skin surface, high degrees of countercurrent exchange result in an increased susceptibility to frostbite. Once again, showing that organisms have evolved to utilize heat exchange between vessels in spite of this demonstrates that thermal efficiency of the body outweighs the potential risks and damages of frostbite in the digits. Succinctly, hypothermia is a far greater concern than frostbite, so the maintenance of core body temperature takes precedence over that of the extremities.

By better understanding the physics behind this process, we can better understand the conditions under which organisms evolved. Furthermore, by better documenting the sensitivities of heat loss to different parameters, we can better understand the criteria by which anatomy is designed.

10 Limitations

The model that was used in this study is an oversimplification of the mechanisms of heat transfer in the finger. In reality, vasculature is complex and non-homogeneous, and temperature profiles are subsequently far more even than suggested by our computations. For the purposes of this study, however, this simplified geometry is sufficient because the focus is on the impact of different variables on heat loss rather than the temperature at individual loci and times in the domain.

For the purposes of sensitivity analysis, it is far more practical to utilize a simplified geometry as it is easier to eliminate confounding factors that arise from manipulations of geometry, as described in Figures 7(b) and 11(b). Furthermore, it preserves the mechanism that we are trying to understand which is countercurrent heat exchange between the major blood vessels of the finger. Thus, for the purposes of this study, this simple model is an appropriate design.

One significant limitation of this model is that it does not mimic the same energy conservation charac-

teristics of the physical system. Since from a computational and model-building standpoint it is impossible to break the arterial blood flow down to capillaries and build it back up into veins, we were forced to specify an inlet temperature that is a simple average of points along the tip of the finger. While this is a reasonable assumption to make since blood residence time in capillaries is sufficient for it to reach steady state with the surrounding tissues, it does not preserve the same conservation of heat. Thus, it is fair to draw conclusions about things like the convective heat flux, or temperature, but we cannot fall back on the heat transfer trope that "in" minus "out" plus "generation" equals change in storage in terms of heat content of the system.

If we were to further study this process, a superior model would incorporate contiguous flow such that heat loss could be expressed as a function of decrease in heat content. Specifically, we would like to be able to express the heat conserved by the system as a function of outlet vein temperature. In our model, we cannot do this, since it was designed with convective heat flux in mind, using an approximation as the venous inlet temperature. For more developed results, a model that allowed blood temperature to equilibrate with tissue temperature in a single contiguous flow is needed.

11 Conclusion

We thus conclude that maximization of countercurrent heat exchange is a critical factor in determining blood vessel geometry in the finger. Relative to benchmarks of a minor increases in heat retention, shown by the modeled effects of vasodilation, vasoconstriction, and changes in blood velocity, distance between vessels proved to be by far the most important determinant of convective heat loss. Since mechanistically, the amount of heat exchanged between vessels is a function of their distance, this suggests that, were vessels located further apart from one another, thermal efficiency of the body would be far less than it is currently.

An interesting extension of this study would be to consider the role of vessel positioning in the temperature profile of the finger using a model more representative of the true thermodynamic properties of blood vessel geometry and flow. Specifically, considering the effect of vessel locations relative to other vessels, bone, etc. on the view factors between said objects could lend a degree of sophistication to the results presented by this study (see Appendix C for more details).

In accordance with our goal of better understanding the mechanistic underpinnings of biological heat conservation, this study is successful in determining the relative importance of the parameters considered. By conducting this study, however, we learned that the process we modeled is far more complex than had been previously suggested. This model provides a basis for bioinspired designs, and a more thorough understanding of the impacts of other parameters such as the view factors between vessels, bone, etc. would help to create a superior database from which to prototype.

Appendices

A Input Parameters

Table A1: Input Parameters and Variable Definitions

Parameter	Symbol	Finger Tissue	Blood	Units
Thermal Conductivity ¹¹	k	0.35	0.45	$\frac{W}{m \cdot K}$
Density ¹¹	ρ	1060	1060	$\frac{kg}{m^3}$
Specific Heat ¹¹	C_p	3900	3900	$\frac{J}{kgK}$
Viscosity ⁶	μ	–	0.0028	$Pa \cdot s$

Table A2: Input Parameter Values

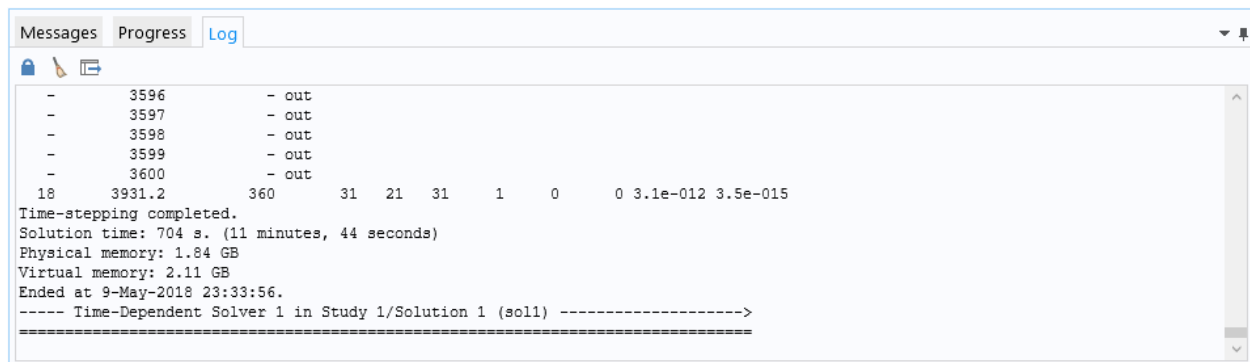
Parameter	Symbol	Value	Units
Ambient Temperature	T_{ambient}	0	$^{\circ}C$
Initial Tissue Temperature	$T_{\text{initial, tissue}}$	37	$^{\circ}C$
Initial Blood Temperature	$T_{\text{initial, blood}}$	37	$^{\circ}C$
Artery Inlet Temperature	$T_{\text{inlet, arterial}}$	37	$^{\circ}C$
Vein Inlet Temperature	$T_{\text{inlet, venous}}$	–	$^{\circ}C$
Arterial Blood Velocity ²	u_a	10	$\frac{cm}{s}$
Venous Blood Velocity ²	u_v	4	$\frac{cm}{s}$
Finger Radius ⁹	R	7.5	mm
Finger Length ⁹	l	80	mm
Absolute Arterial Pressure ⁶	P_a	1.1597	atm
Absolute Venous Pressure ⁶	P_v	1.1053	atm

B Solution Strategy

```
===== Opened 3D_Countercurrent_009.mph =====  
Number of vertex elements: 24  
Number of edge elements: 2118  
Number of boundary elements: 25,992  
Number of elements: 410,059  
Free meshing time: 6.03s  
Minimum element quality: 0.1922  
=====
```

Solution time: **704 s.** (11 minutes, 44 seconds)
Physical memory: **1.84 GB**
Virtual memory: **2.11 GB**
Ended at 9-May-2018 23:33:56.
----- Time-Dependent Solver 1 in Study 1/Solution 1 (sol1) ----->

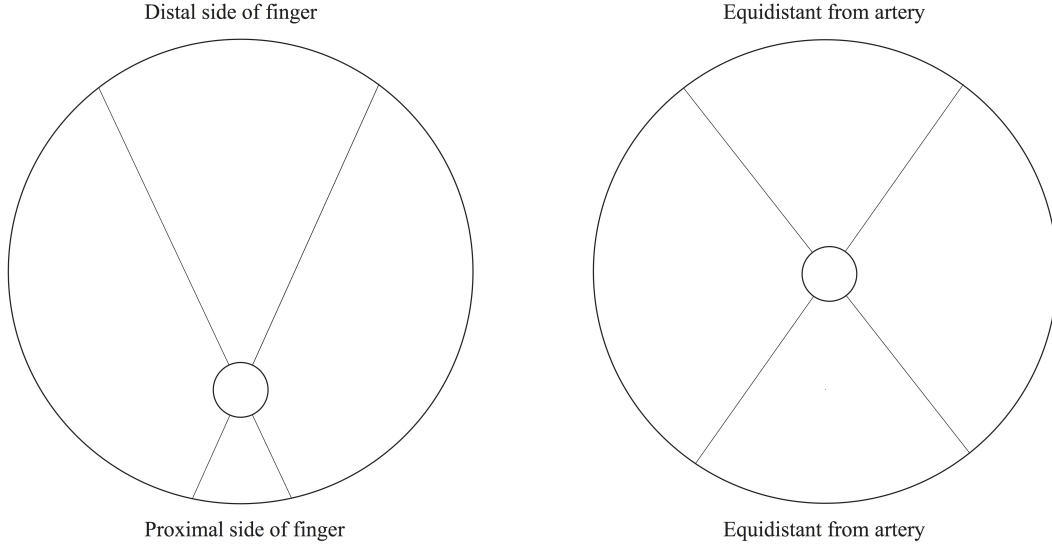
Figure B1: Total of 410,059 elements and a total solution time of 704 seconds (11 minutes and 44 seconds) for a study time of 3600 seconds. The model utilized 1.84 GB of physical memory and 2.11 GB of virtual memory.



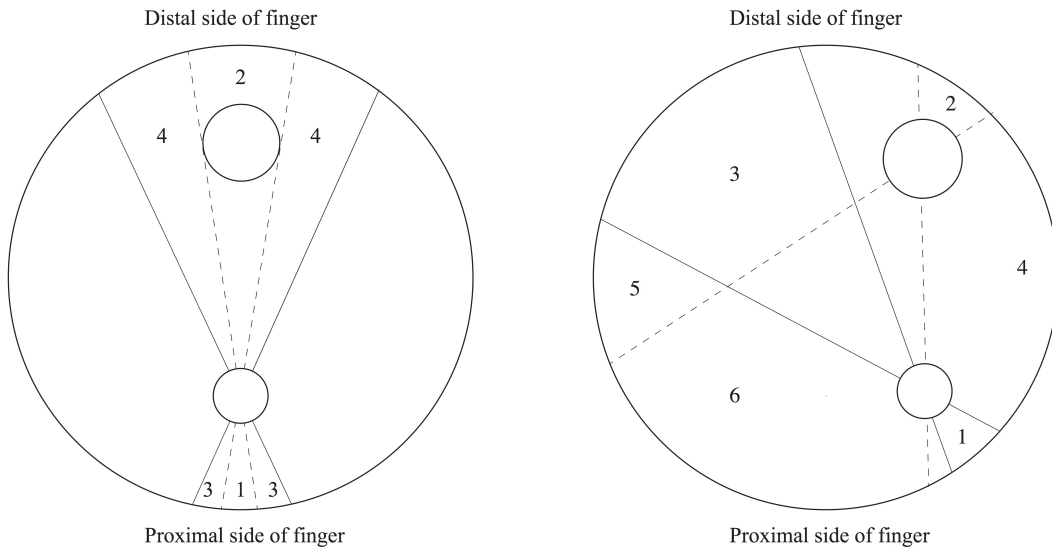
```
Messages Progress Log  
- 3596 - out  
- 3597 - out  
- 3598 - out  
- 3599 - out  
- 3600 - out  
18 3931.2 360 31 21 31 1 0 0 3.1e-012 3.5e-015  
Time-stepping completed.  
Solution time: 704 s. (11 minutes, 44 seconds)  
Physical memory: 1.84 GB  
Virtual memory: 2.11 GB  
Ended at 9-May-2018 23:33:56.  
----- Time-Dependent Solver 1 in Study 1/Solution 1 (sol1) ----->
```

Figure B2: The CPU time taken and the memory being used by a typical study time of 3600 seconds.

C View Factor Schematics



(a) Schematic showing the view factor of an artery with proximal and distal sides of the finger (b) Schematic showing the view factor of an artery positioned at the geometric center of the finger



(c) Schematic showing the view factor of vessels positioned centrally (d) Schematic showing the view factor of vessels displaced towards surface

Figure C1: Schematic showing the view factor of vessels in different geometries. Numbers are assigned for reference to regions of the surface in future discussion. Geometries (c) and (d) were chosen to represent the configurations that yielded surprising results in the sensitivity analysis on displacement.

The concept of view factor in the context of vessel location in the domain of the finger is used to explain the unexpected conclusion that convective heat flux is actually decreased if vessels are skewed to one side. Figure C1(a) and (b) demonstrate the negligible effect of skewing a single vessel to one side of the geometry. While there is a larger increase in surface temperature along the proximal side of the finger when the vessel is displaced, this is offset by the increase in surface area of the distal side, which receives the least heat from the artery. We would thus expect to see no appreciable difference in average surface temperature (of which convective heat flux is a function) as a result of this displacement. This statement was validated using a simple two dimensional COMSOL model in of the two concentric circles shown in Figure C1(a) and C1(b) with a temperature specified at the boundary of the inner circle and a convective heat flux condition specified at the surface of the larger circle. There was no difference at all in the surface temperature of the larger circle.

This problem becomes more complicated if two vessels are considered. In Figure C1(a), the distal side of the finger receives very little heat from the warm arterial blood. in Figure C1(c), the placement of a vein in the model creates a warm penumbra (region 2) at the distal surface of the finger, effectively warming what would otherwise be the coldest region of the domain. Region 4 sees this effect to a lesser degree, and regions 1 and two remain hot as in the original scenario. In contrast, if the vessels are displaced as shown in Figure C1(b), the same warming effect is not observed. Instead, the region that is most warmed by the venous blood (region 2) lies outside of cool distal side of the finger. Since the skewed geometry features warming of a region that is warmer to begin with, it decreases the net effect of the venous blood on the warming of the finger.

Ostensibly, the heat loss of a cylinder containing two countercurrent fluid vessels is governed by the net sum of the view factor effect described above, and the decrease in insulation at the closest surface. The results of our modeling suggest that the view factor effect is more significant. This is perhaps the most novel aspect of this study; however, as it is not the original focus, there is not conclusive data to support it. It would be an interesting subject of future study to consider the effects of tweaking vessels to improve this "penumbric shielding" as a means of conserving heat.

D References

- [1] Andreas D. Flouris et al. “Effect of body temperature on cold induced vasodilation”. In: *European Journal of Applied Physiology* 104.3 (Oct. 2008), pp. 491–499. DOI: 10.1007/s00421-008-0798-3.
- [2] M. Klarhöfer et al. “High-resolution blood flow velocity measurements in the human finger”. In: *Magnetic Resonance in Medicine* 45.4 (Apr. 2001), pp. 716–719.
- [3] Joan McMeeken. “Tissue temperature and blood flow: a research based overview of electrophysical modalities”. In: *Australian Journal of Physiotherapy* 40 (1994), pp. 49–57. DOI: 10.1016/S0004-9514(14)60623-1.
- [4] John Mitchell and Glen Myers. *An Analytical Model of the Counter-Current Heat Exchange Phenomena*.
- [5] Douglas O. Nelson, James E. Heath, and C. Ladd Prosser. “Evolution of Temperature Regulatory Mechanisms”. In: *American Zoologist* 24.3 (1984), pp. 791–807.
- [6] G. Parati et al. “Comparison of finger and intra-arterial blood pressure monitoring at rest and during laboratory testing.” In: *Hypertension* 13.6 (June 1, 1989), pp. 647–655. DOI: 10.1161/01.HYP.13.6.647.
- [7] Christiane Pfarrer et al. “The three-dimensional feto-maternal vascular interrelationship during early bovine placental development: a scanning electron microscopical study”. In: *Journal of Anatomy* 198 (Pt 5 May 2001), pp. 591–602. DOI: 10.1046/j.1469-7580.2001.19850591.x.
- [8] Loring B. Rowell. “The Cardiovascular System and Heat Transfer”. In: *Heart Perfusion, Energetics, and Ischemia*. NATO Advanced Science Institutes Series. Springer, Boston, MA, 1983, pp. 509–538. DOI: 10.1007/978-1-4757-0393-1_25.
- [9] Avraham Shitzer et al. “Numerical analysis of an extremity in a cold environment including counter-current arterio-venous heat exchange”. In: *Journal of Biomechanical Engineering* 119.2 (May 1997), pp. 179–186.
- [10] Avraham Shitzer et al. “Simultaneous measurements of finger-tip temperatures and blood perfusion rates in a cold environment”. In: *Journal of Thermal Biology* 22 (June 1, 1997), pp. 159–167. DOI: 10.1016/S0306-4565(97)00004-1.
- [11] Yan Zhang and Haiwei Xie. “The effect of a bifurcation structure on the heat transfer and temperature distribution of pulsatile blood flow”. In: *International Journal of Heat and Mass Transfer* 118 (Mar. 1, 2018), pp. 663–670. DOI: 10.1016/j.ijheatmasstransfer.2017.11.055.

- [12] L. Zhu. “Theoretical evaluation of contributions of heat conduction and countercurrent heat exchange in selective brain cooling in humans”. In: *Annals of Biomedical Engineering* 28.3 (Mar. 2000), pp. 269–277.